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What Is a Thing?

I am not going to comment on Heidegger's *Die Frage nach dem Ding*: I lack the skills and education for that. The reason for choosing such a title is that Heidegger's views on theoretical physics, as they are expressed in this series of lectures delivered in 1935-36, seem to me the best way to introduce the subject of my talk the aim of which is to describe how contemporary physics (meaning physics in the last ~~five~~ fifty years, from the 1960s on) deals with the eternally reworked concept of *res* (*chose*, thing, *Ding*)

Let me briefly sketch out what Heidegger says. Looking for what can be considered specific to modern science (i.e. post- Galilean science), Heidegger finds it in its mathematical character – which looks like a common statement, but appears more complicated and sophisticated after closer examination. More precisely, although Heidegger (in *Sein und Zeit*, 1929) had already described modern science as a “mathematical sketch or draft (*Entwurf*) of nature,” he now (that is, in 1935, after some rather rough discussions with Cassirer) realizes that there is more to it: the fundamental characteristic of modern science, that which distinguishes it from both ancient and medieval science, is neither its claim to deal with facts, nor its emphasis on experiment and numerical measurements, but rather the discovery that mathematics has something to do with things, that mathematics “meets” things : “Mathematics is concerned with things, although from a specific perspective” [*Das Mathematische betrifft die Dinge und zwar in einer bestimmten Hinsicht*]¹

¹ . Martin Heidegger, *Die Frage nach dem Ding*, Tübingen: Max Niemeyer Verlag, 1962, 54. Martin Heidegger, *What is a Thing?* Trans. Deutsch, Barton, Gendlin. New York: Gateway, 1968, 70.

Heidegger's view cannot, and must not, be reduced to the usual statement that modern science aims at a "mathematization of Nature". As a matter of fact, the expression "mathematization of Nature" can be given at least two different meanings. A first meaning relies on the idea of a *representation*: mathematics (through theoretical physics) gives a picture of the "real" world – an idea that needs to be complemented by a statement of what kind of picture is meant. For Heinrich Hertz (1857-1894) for example, whose work was very influential at that time, the idea is that we form, not exactly pictures, *Bilder*, but simulacra, (*Scheinbilder*) of things (*Gegenstände*), such that the relations between, let us say two, *Scheinbilder* (or *Symbole*), reproduce the relations among the things they represent.² This first meaning of mathematization as representation is the one advocated by Cassirer: it implies that there exists such a thing as a "mathematical science of Nature" in which Nature itself is symbolically depicted. This is not Heidegger's view. Heidegger has a more profound understanding of the way mathematics intervenes in physics, and thus in Nature ; things -- he says -- are not represented by mathematics ; they are "viewed and spoken of mathematically" [*mathematisch angesehen und angesprochen*].

The question then is: what is the perspective from which they are thus *genommen*, "grasped" or "seized." "In what respect are things taken when they are viewed and spoken of mathematically?" [*In welcher Hinsicht sind die Dinge genommen, wenn Sie mathematisch angesehen und angesprochen werden?*]³ asks Heidegger. Nature, it would seem, vanishes. Nature is no longer the collection of things (or phenomena) involved in the previous meaning of "mathematization" and knowing is not taking a picture, or a photograph which reproduces the relations among things. According to Heidegger's view of theoretical physics, a *perspective* or *Hinsicht* is introduced, with the obvious drawback of thereby introducing a distortion (especially as relations among things are concerned), since a perspective is in itself a distortion. Sure enough, there is such a risk but, says Heidegger, it is worth taking; in fact, we

² The metaphor is that of a token which stands for a coat in a cloakroom.

³ *Frage*, 54; *What is a Thing*, 70.

cannot do otherwise. For mathematics touches upon things and this *must* be taken into account, no matter what the drawbacks.

So we do not see Nature “as it is” but from the point of view of mathematics. Nature itself has been replaced by “the book of Nature, written .as everybody knows (see Galileo) in “characters [which] are triangles, circles, and other geometrical figures.”⁴ And here, the whole passage from *Il saggiatore* should be quoted :

“Philosophy is written in this vast book, which lies continuously open before our eyes (I mean the universe). But it cannot be understood unless you have first learned to understand the language and recognize the characters in which it is written. It is written in the language of mathematics, and the characters are triangles, circles, and other geometrical figures. Without such means, it is impossible for us humans to understand a word of it, and to be without them is to wander around in vain through a dark labyrinth.”

Galileo does insist repeatedly on the fact that we are impelled in that direction, and that there is no other way. In the *Dialogo* he argues that “It must be admitted that he who undertakes to deal with questions of natural sciences without the help of geometry is attempting the unfeasible.”

So the question Heidegger is asking becomes what does reading the book of Nature (and not Nature itself), written in geometrical language, imply? What does it change? The answer is clear and simple: geometry introduces *space*. Things, as we “see” them through the perspective of mathematics, no longer exist by themselves, they are part of a geometrical construction. They are not *vorhanden* (present-at-hand), they are *zuhanden* (ready- to- hand). Things are, so to speak, embedded in space; they cannot be disentangled from space. Or, in other words, things are “grasped” by mathematics, using the tool of space to grasp them – just as we grasp sugar cubes with a pair of tongs. Or, more appropriately, they are “caught” in the net of space, like an insect in a spider web. Although all metaphors are deficient, I

⁴ Galileo, *Opere* 6:232.

think this last one is not too bad, for, as we shall see in a moment, a geometrical space is mathematically equivalent to a net of lines, called geodesics, of fundamental importance to theoretical physics. In Heidegger's words: "Nature is no longer the inner principle out of which the motion of the body follows. Rather, nature is the mode of the variety of the changing relative positions of bodies, the manner in which they are present in space and time, which themselves are domains of possible positional orders and determinations of order and have no special traits anywhere." [*Natur ist jetzt nicht mehr das, was als inneres Vermögen des Körpers diesem die Bewegungsform und sein Ort bestimmt. Natur ist jetzt der im axiomatischen Entwurf umrissene Bereich des gleichmässigen raumzeitlichen Bewegung Zusammenhanges, in der eingefügt und verspannt die Körper allein Körper sein können*].⁵

What does Heidegger mean when he says that bodies, once caught in space (*eingefügt und verspannt*), can only be bodies (*allein Körper sein können*)? Obviously "Körper sein," (or "be bodies") refers to the substantial qualities of things. Heidegger's "allein" ("alone" or "only") seems to imply that, once they are caught in the array of space, things are free to become only corporeal, substantive, material. They are, so to speak, stripped of their other properties. How are we to understand this? Because, from Galileo's time up to 1850, the space in which things are caught could only be Euclidean space, that is, homogeneous and isotropic space, we are led to the conclusion that through this process of capture, the qualities of things, which a priori must be an intricate combination of spatiotemporal and substantial properties, appear, once they have been processed by geometry (as they are in the mathematical *Entwurf*), as disentangled and divided into two categories: spatial properties on one hand, substantive properties on the other. In contrast to the Aristotelian conception of the world where substantive properties of things were supposed to determine (or influence) their localization, once things are caught in homogeneous and isotropic Euclidean space, they become something else -- an

⁵ *Frage, 72, What is a Thing, 88.* A more accurate translation would be: "Nature is no longer that which, as inner capacity of the body, determines the form of its movement and its location. Nature is now the domain whose contours are outlined in the axiomatic project, a domain of the uniform spatiotemporal connection of movements such that the bodies inserted it can only be bodies."

object (*ob-jectum*) says Heidegger --something to which substantial properties can be attributed. The question then arises of how this can take place. In classical physics (post-Galilean but pre-Einsteinian), it is done in the simplest manner by ascribing one or more numerical coefficients to each of these objects (geometrical objects, i.e., the points or set of points of Euclidean geometry) which are supposed to encompass all spatiotemporal properties of things. Among such coefficients, mass is the most common: it is supposed to take into account the fact that the thing considered is material. That this way of representing material properties of things is too simple a procedure is suggested by the fact that, at some point, physicists came to realize that two (and not one) coefficients need to be defined in order to fully account for the material properties of things: one for inertial properties (the resistance of matter to change of motion) and one for gravitational properties (matter as such attracts matter). Among other substantial coefficients, charge (electric charge) and potential (a quantity from which the forces acting on a “thing” can be deduced) are also of importance.

Heidegger then concludes that “Natural bodies are now only what they show themselves as, within this projected realm. Things now show themselves only in the relations of places and time points and in the measures of mass and working forces” [*Die Naturkörper sind nur das, als was sie (the things) sich im Bereich des Entwurfs zeigen. Die Dinge zeigen sich jetzt nur in der Verhältnissen der Örter und Zeitpunkte (spatiotemporal properties), und den Maßen der Masse und der wirkenden Kräfte (material qualities).*]⁶

Note the “*nur*” (*die Dinge sind nur das*—Natural bodies or things are now only what--) which suggests that things are *reduced* to objects to which coefficients are attached. This is no surprise since, as noted above, this procedure corresponds to a certain inevitably distorted perspective. “What remains questionable in all this is a closer determination of the relation of the mathematical in the sense of mathematics to the intuitive direct perceptual experience of the given things and to

⁶ *Frage. 72 What is a Thing*, 93. Heidegger refers here to forces acting on the object, which physicists generally take into account by means of what they call a “potential”.

these things themselves” [*Fraglich bleibt dabei, die nähere Bestimmung des verhältnisses des mathematisches im Sinne der Mathematik zur anschaulichen Erfahrung der gegebene Dinge und zu diesen selbst*].⁷ One might even think that things, rather than being incompletely described by this treatment, come out of it severely injured – even dead, why not? Which recalls Count Zaroff’s famous reply (in the 1932 film adaptation of “The Most Dangerous Game”): “First kill, then love.” In a similar way, the motto of classical physics, as far as bodies are concerned, could be: Give them a mathematical existence (kill them as things), then make the corpses look like bodies by decorating them – so that you can interact with them as if they were things.

That is how we feel today; we have no problem with the fact that catching things by means of a geometrical perspective produces a distortion. But for more than two centuries, space was not a problem. Time even less. Space was conceived as a kind of “vanishing mediator” (a special interpretation of that famous expression taken out of its original context), an ideal pair of tongs which catches things without disturbing them in any way. As Heidegger says space was supposed to be a perfect tool: the mathematical, is, as mental conception, a project (*Entwurf*) of thingness (*Dingheit* which, as it were, skips over the things. [*Das Mathematische ist, als mente concipere, ein über die Dinge gleichsam hinweg springender Entwurf ihrer Dingheit*].⁸

What does Heidegger mean? What is thingness (*Dingheit*)? (in French *Dingheit* is translated by *choséité*)? And how is the skipping metaphor (*hinweg springend*) to be understood? Why is Euclidean space so suitable for grasping the *Dingheit* of things?

⁷ *Frage*, 73, *What is a Thing*, 93-94.

Space

To respond requires a closer examination of the role of space.

A major change in theoretical physics was introduced by Riemann's *Habilitationsvortrag*, written in 1854, issued in 1867, "*Über die Hypothesen, welche der Geometrie zu Grunde liegen*" (On the Hypotheses that Lie at the Foundation of Geometry).⁹ One usually thinks of Riemann as the mathematician who revolutionized geometry, making it possible for non-Euclidian geometries to develop. This is an incomplete and partial view. For classical physics being what it was in the middle of the nineteenth century --i.e., based on the above description of things as caught in Euclidean space -- a revolution in geometry necessarily meant a revolution in physics: both the objectivity of the latter (physics) and the validity of its proofs carried on "*more geometrico*" had relied on the former (geometry). As is documented in his private papers, Riemann's motivations in writing his *Habilitationsvortrag* were, as we know now, related to his concern about physics. More precisely, Riemann was trying to construct a mathematical concept of field, a task James Clerk Maxwell was also aiming at, more or less at the same time.

Being an adept of the so-called philosophy of nature, Riemann denied the Kantian idea of space as a *given* entity, a "pure intuition of the mind," and attempted to mathematically construct the concept of space: "It is known that geometry assumes, as things given, both the notion of space and the first principles of constructions in space" [*Bekantlich setzt die Geometrie, sowohl den Begriff des Raumes als die ersten Grundbegriffe für die Constructionen im Raume, als etwas Gegebenes voraus*].¹⁰ Building a concept of space from scratch is achieved in two steps. First, Riemann invents the general notion of a "multiply extended magnitude" (which he calls a "manifold," *Mannigfaltigkeit*), then, he asks the question: how can physical space (as the set of possible positions of objects and therefore things) be characterized as a *special case* of such a general magnitude? He sees this selection process, as an

⁹ Bernhard Riemann, *On the Hypotheses that Lie at the Foundation of Geometry*.

< <http://www.emis.de/classics/Riemann/WKCCGeom.pdf> >

¹⁰ Riemann, p. 1.

iterative process of specification based on experience (*Erfahrung*), i.e., he singles out some observed regularities (he calls them “*Tatsachen*”, translated into English as “matters of facts”) displayed by physical space, which will determine (i.e. mathematically specify) physical space. This specification is achieved in successive steps, each “matter of fact” implying a specific restriction or selection among all possible manifolds, out of which another “matter of fact” will select a more restricted class of manifolds, and so on, up to the end, that is a full determination (*eindeutig Bestimmung*). Physical space is thus a work in progress: it is *constructed* by adding successive hypotheses of empirical origin, each of which implies a mathematical restriction on the general concept of manifold.

Let me emphasize how far we are from the idea of a *given* space, Euclidean space, as Kant describes it. Riemann is *actively structuring* physical space as a specific manifold by focusing on some key *observed* properties which are turned into “added hypotheses” (thus, the title of his *Habilitationsvortrag: Über die Hypothesen, welche der Geometrie zu Grunde liegen*). The task then consists in making these regularities explicit. With the proviso that “These matters of fact are—like all matters of fact—not necessary, but only of empirical certainty; they are hypotheses” [(*Diese Tatsachen sind wie alle Tatsachen nicht notwendig, sondern nur von empirische Gewissheit, sie sind Hypothesen*)].¹¹

What becomes of things if such a view of space is adopted? Remember that classical physics (from Galileo on) relies on the seizure of things by space (meaning a *given* entity, the geometry of which had been spontaneously identified with Euclidean geometry. Physics had relied for its objectivity and certainty on such an identification: planes in physical space, just to mention one instance, were implicitly supposed to be perfect Euclidean planes, perfectly flat. What happens if it is found that physical planes are not ideally plane, but curved (a question Riemann inherited from Gauss who had already empirically investigated it). One would expect that

¹¹Riemann, 14.

Riemann's destruction of Euclidean space as an absolute source of certainty would result in invalidating the whole project (*Entwurf*) of mathematical physics as it had been carried out up to then. In particular, what would happen to the transformation of things into geometrical objects, which had been so efficient in separating substantial characteristics of things from other qualities?

At this point, in order to give a better account of how catastrophic the situation was as far as physics was concerned, I must add something to my previous account of Riemann's *Habilitationsvortrag*, something I had postponed for the sake of clarity. This something deals with the concept of distance between two points in a given space, what mathematicians call its metrics. In Euclidean space, such a quantity is absolute, meaning independent of the positions of the two *points* in space, invariant under a shift of the two points from here to there. In the project of mathematical physics, in its original form, this property of invariance is transferred to the distance separating two "things" This is possible because the substantial properties of things are disconnected from their spatial locations. But, if physical space is not identified with Euclidean space (and its geometry), there is no reason why this should still be the case. In fact, as Riemann has shown in its *Habilitationsvortrag*, the distance between two "points" of a manifold is generally *not* independent of their (global) position in space; it depends on what Riemann calls "the binding forces which act upon space" (*wirkende bindende Kräfte*),¹² something which is not included in the concept of space, something outside of it (*ausserhalb*). Applied to physical space, to that which accommodates things), this means that the very disconnection of substantial from spatial properties of things might become problematic. The whole enterprise of mathematical physics seems to fall apart.

Equivalence

¹² Riemann, p. 17. Once more Riemann and Heidegger use the same words

I am now going to explain how the mathematical *Entwurf* or project, which really got into trouble when it was realized that space was not what people had thought it was, got out of this trouble. As we shall see, it did so only by rushing headlong into more abstract mathematics.

“Equivalence” is the key to the way out, as physicists came to realize many years later, in the second half of the twentieth century. Explaining how the concept of equivalence was introduced will give the answer to the questions I previously asked when examining Heidegger’s account of the role played by mathematics in physics: 1) What does he mean by *Dingheit*? and 2) How is it that Euclidean space is (or was) particularly suitable for grasping things in their *Dingheit*?

Since this is not so easy to articulate, I am going to proceed in steps, which carries the risk of being too formal, or too military.

1. *Res*

As we know, the word “thing” (“*Ding*”, “*chose*”, “*cosa*” in modern European languages) is the standard translation for the Latin word “*res*”. One of the characteristics of this word, much commented upon, is that it is ambiguous and indeterminate and that this semantic indeterminacy explains why the notion has become so important. Although this indeterminacy has many sides, one of them has played a decisive role in the history of mathematics, namely the fact that what is implied in the word “*res*” is at the same time definite and indefinite, individual and general – which Avicenna (Ibn-Sina) explains using the example of the horse: “in itself *equinitas* [meaning “horseness,” derived from *equinus*] is neither one nor many.”¹³ This is the first point: *things are neither one nor many*.

Let me just open a brief parentheses before I proceed to the second point: it has been proposed¹⁴ that the kind of ontology implied by the example of *equinitas*

¹³ “*Unde ipsa equinitas non est aliud nisi equinitas tantum; ipas enim in se nec est multa nec est unum*” Avicenna latinus, *Liber de philosophia prima, sive scientia divina*, 3 éd. S. van Riet, 228, 229-36, t. 1, 234, Louvain-Leyde : Brill, 1977-1983. Cited in the entry « *Res* » *Vocabulaire Européen des Philosophies*, B. Cassin (dir), Paris : Editions du Seuil/Le Robert, 2004.

¹⁴ See R. Rashed, “Mathématique et philosophie chez Avicenne”, in J. Jolivet, R. Rashed, *Etudes sur Avicenne*, Paris, Les Belles Lettres, 1984, 29-35.

(an ontology expressed in the term *shay'yya* or thingness in Arabic) was at the root of algebra, which developed independently of Greek epistemology in the Arabic word, as a science common to both arithmetic and geometry, a science where the unknown “x” can be either a number or a geometrical magnitude. I mention this just to suggest that this might contribute to answering the unanswered question: how is it that physics which started as geometric ended as algebraic.

2. What is so special about Euclidean space?

I come to my second point. Although this is not a common view on Euclidean space, one must realize that being *one and many* (or rather neither one nor many) is precisely what characterizes points of Euclidean space. In Euclidean space, “Here” is nothing by itself that might differ from any other “Here”; all “Heres” are equivalent, *not identical but equivalent*.

Suppose I ask you the question: how do you define a point in Euclidean space? Having gone to school and studied elementary geometry, you will probably give the right answer: “I choose a reference system, meaning three perpendicular axes along which coordinates (Cartesian coordinates) can be measured; then I ascribe numerical values to these three coordinates; they uniquely determine a point.” Okay. But are you sure the point thus arrived at is uniquely determined? No it is not. You (rightly) said: I first *choose* a reference system. An entirely free choice. You could have chosen any other frame of reference; it would serve equally well. That is what I mean when I say points in Euclidean space are all equivalent. They are at the same time one (I can put my finger on it) and many (they are not uniquely determined), neither really one nor really many.

This is of paramount importance. For it is precisely what makes the mathematical *Entwurf*, as it is described by Heidegger, feasible. With a same “thing” (with a set of given substantial coefficients) different equivalent points (spatial locations) may be associated – a state of affairs that is usually referred to as the relativity of positions. That the point associated with a given “thing” is not

unique but part of a class of equivalent points is what allows the connection of “things” to points in Euclidean space. Points in Euclidean space share with “things” the faculty of being at the same time *singular and multiple*, “one” and “many.” To both of them, “things” and points in Euclidean space, can be attributed the property Avicenna calls thingness. There lies -- in the fact that things and points in Euclidean space enjoy the same thingness,¹⁵ -- the effectiveness of Euclidean geometry as a tool for changing things into objects, the faculty of jumping over things themselves in direction of their thingness or *Dingheit*. *Dingheit* is just that: the property of being one and many. This I identify with equivalence.

3. Equivalence regained

I now come to my third point. Euclidian space is, as Riemann has shown, a very special case of space: homogeneous, isotropic, with zero curvature and so on. The question then is whether equivalence -- that property of points that allows one to grasp directly the fact that things are at the same time one and many -- specific to Euclidean space? In such a case, the mathematical *Entwurf* would be hopeless. Unless the equivalence of points can be regained within the framework of the Riemannian concept of space.

Let us look at equivalence in more detail. Equivalence can be given a precise mathematical treatment in terms of “transformations”, i.e. one-to-one mappings. When we say that points in Euclidian space are all equivalent, we mean that, given any two points, A and B, there always is a point transformation carrying A into B which leaves the basic relations (continuity, connectivity, metric and so on) of Euclidean space unchanged. Since A and B are any two points (in Euclidean space), this means that *in that case* there exists a lot (an infinity) of transformations (translations, rotations, in fact all possible geometrical “displacements”) that transform Euclidean space in itself, or as we say, leave it *invariant* (we start with Euclidian space and the result is Euclidean space itself).

¹⁵ (Note that neither points in Euclidean space nor things are completely determined by this property).

There is no reason why the notion of invariance, should be restricted to Euclidean space. All the more, since physical space according to Riemann is not given but must be constructed, which implies that we can choose to construct it in such a way that invariance is structurally included in its construction. Now, physical space, in Riemann's view, is constructed by adding hypotheses, each one originating in a selected "matter of fact," *Tatsache*, (selected for its empirical certainty). What characterizes such an empirically certain "matter of fact" is that it can be observed (*wahrgenommen*), meaning that, under certain conditions (those of observation), it is unchanged. Which, in mathematical terms, amounts to requiring that certain transformations that bring physical space onto itself leave that "matter of fact" unchanged, invariant. It can be shown that these transformations form what is called a group, in the mathematical sense of the word: two such transformations amount to a transformation of the same kind; to each transformation can be associated its inverse; there exists a transformation which changes nothing: identity. We come to the conclusion that to each *Tatsache* to be considered when following Riemann's prescription for constructing physical space, there corresponds a group of transformations that leave the space being built invariant.

As Hermann Weyl puts it: "We start with a group of transformations. It describes, as it were, to what degree our point field (space) is homogeneous. *Once the group is given, we know what "equivalent" means. ...*"¹⁶ In other words, we know how to connect things to elements (let us call them "points"), having the same thingness faculty of being one and many. "Doing physics" then amounts to looking for relations that are invariant under all transformations of group ; some people might wish to call them "real" (or even "true").

As a consequence, the primary task in theoretical physics is to identify the right space, that is to say, the right group of transformations , and therefore, the right "matter of fact," that is, the one that is determinant. As Riemann pointed out,

¹⁶ Hermann Weyl, *Philosophy of Mathematics and Natural Science*. Trans. Olaf Helmer. Princeton: Princeton University Press, 1949 [1927], 73.

the space thus chosen has no absolute certainty, no necessity. Should we say that it must only be convenient? I am not going to discuss that issue today. I just want to give two examples. In special relativity the relevant space is the one which is invariant under the Lorentz group (Lorentz transformations of Minkowski's spacetime), the matter of fact being the principle of relativity restricted to a class of reference systems (inertial or Galilean reference systems). In general relativity, the space which has been found to be adequate is a Riemannian space whose curvature depends on the distribution of energy – the corresponding “matter of fact” being the generalized principle of relativity.¹⁷ In dealing with Quantum Mechanics, the space which is convenient is the more abstract, Hilbert space, in which some invariances (space, time invariances, the principle of relativity in its special or general form) are imposed. In both cases, one looks for relations (laws) which are invariant under the transformations of the groups which determine this adequate space. In other words: the equivalence of points in space -- the necessary condition for transforming things into geometrical objects and touching upon their *Dingheit*-- is regained, although in a restricted way compared to the general equivalence which structures Euclidean space.

Invariance, symmetry, transformations, group, equivalence and all that

Let me summarize: mathematical physics (as it was launched by Galileo) relies on the possibility of grasping the *Dingheit* of things, (i.e., the fact that a thing, among other properties, is at the same time *one and many*) in the web of a space. I say *a* space although initially the space involved was unique, namely Euclidean, space. We are lucky that the realization (with Riemann) that physical space is not identical with Euclidean space, did not wreck the whole enterprise initiated by Galileo. The reason for this is 1) the realization that the only important characteristics of Euclidean space, as far as the enterprise of physics is concerned,

¹⁷ Weyl, 104.

is that its points are *equivalent* (and thus one and many, the same way things are); only the existence of equivalent points is needed in order to capture the *Dingheit* of things; 2) that such a requirement (the existence of equivalent “points”) is, so to speak, automatically fulfilled if one holds to the Riemannian way of constructing physical space, starting from a choice of empirical data.

As we can see, *equivalence* is the key notion in post-Riemannian physics. Related to it is the notion of *transformation*: objects (points) are equivalent not *in abstracto* but as far as certain transformations of the set they belong to (the space) are concerned. Note that transformation is a notion that was unknown to classical geometry which only considered figures related to one another through kinship: equality, similarity, etc. The notion of transformation changes that of space which is no longer considered a collection of points, but is now seen as a structure that can be applied to itself, and therefore remains *invariant* as a whole through a definite set of transformations.

We owe to Felix Klein and Sophus Lie the realization that transformations can be studied as such: they have shown that two transformations commute, that their combination results in a transformation of the same kind – in other words that the concept of *group* invented 175 years ago by Évariste Galois in an algebraic context, applies to them. *Equivalence, transformation, invariance and group* now form an epistemological constellation. To which it is easy to add another notion: *symmetry*, a common notion¹⁸ which acquires a more precise and mathematical definition involving the terms “transformation” and “invariance.” A symmetry (note the indefinite pronoun “a”) is a transformation that globally leaves invariant the relevant structure.”¹⁹

Today, “symmetry” has become the general word for this constellation: invariance, group of transformations, equivalence.

¹⁸ The word “symmetry” (and even more the adjective “symmetrical”) belong to our ordinary vocabulary; we say that a cube and a sphere are symmetrical. But if you think a while about it you will rapidly see that the common language is not precise enough. For instance a cube may be said to be symmetrical in different ways: by tilting over one of its faces, by turning it upside down. The specification to be added to the word “symmetrical” is always an operation, in other words, a transformation

¹⁹ B. van Frassen, *Laws and symmetries*, Oxford: Oxford University Press, 1989, 388.

The Identity of things: a contemporary perspective

Up to now I have focused on how things are caught by space (a new concept of space) in their *Dingheit*. Once this problem is solved (or satisfactorily solved), there remains the problem of completing their characterization; let us recall that the problem was solved in classical physics by the mere adjunction of “substantial coefficients” to points in space (Euclidean space). How are these characterizations going to evolve in the new context of symmetry?

The answer relies on two fundamental theorems due to Emmy Noether published in 1918.²⁰ Emmy Noether has shown that for a system exhibiting a certain group of symmetry, to each of the transformations of that group, there corresponds a quantity which is invariant – or, in physical terms, there corresponds a magnitude relative to the system that retains the same value under the transformations of the group. In other words, a certain physical quantity (or magnitude) is “conserved” when the system itself undergoes the corresponding changes. More practically, this implies that these quantities can be observed, and measured. This is the reason why they are very often called “observables”.

This will be more understandable if I give an example. Consider the symmetry called “invariance under space translation” which is the way the hypotheses of homogeneity of space (things behave the same way here as in Tokyo or on the Moon) is translated into the language of symmetries and invariance, the new language of physics. According to Noether’s theorem, to that invariance there corresponds a conserved magnitude for any system, anything placed in such a homogeneous space. It can be shown that this magnitude which keeps the same value under any change of the thing in that homogeneous space, is what is called impulsion, or linear momentum, in classical mechanics. As in the case of the supposed isotropy of space, there corresponds the fact that, for any system imbedded in such a space, the magnitude which is usually called “angular

²⁰ E. Noether, “Invariante Variationsprobleme,” *Königliche Gesellschaft der Wissenschaften Nachrichten. Math.-Phys. Klasse*, n°2 1918, 235.

momentum” is conserved, keeps the same value while the thing itself undergoes changes in that space. Such magnitudes which keep a constant value while the thing undergoes changes can rightly be considered as part of the identity of the thing, its ID.

The same can be said for less intuitive symmetries, like for instance the invariance which is called “gauge invariance” to which there corresponds a conservation of electric charge. This last example is instructive since it makes it obvious that what used to be called a substantial characteristic in the former view of things, now comes out of an invariance (or symmetry) property. The same holds if we now consider, not only continuous symmetries, but also discrete symmetries, such as parity (equivalence in mirror image, or change in the directions of space), inversion of time, charge conjugation (going from matter to anti-matter), etc.

It should be noted that all these ID characteristics are related to things as being imbedded in (a) space, not things in themselves – which we know has no place in the mathematical *Entwurf* launched in the seventeenth century. As you probably know, the main problem facing theoretical physics today is how to unify quantum mechanics and general relativity (in other words, gravitation theory). A unification along the lines I have just sketched out, i.e. based on Finding a general group of symmetries the sub-groups of which would include quantum mechanics (I should say mechanics, because one of the most impressive results of this approach is that the same symmetries hold for quantum and classical physics) and general relativity. String theory (or supersymmetric strings) seems to be the best way to achieve such a unification. But...

In this talk, I wanted to discuss two points which I think are not sufficiently commented on outside the cenacle of mathematicians and physicists.

- 1) space (whatever be its definition) is a non-avoidable ingredient of phenomena; they occur in space; things are embedded in space; there is no way to ignore it. Riemann was a pioneer in the investigation of how

the behavior of things (matters of facts, phenomena) and the geometrical structure of space are intertwined;

- 2) group theory, because it touches upon both geometry and arithmetic (through the concept of conservation, as investigated by E. Noether), has replaced (and complicated) the classical grasp of things by means of Euclidean space and arithmetic substantial coefficients.

Paraphrasing the title given by one of my colleagues, Francis Bailly, who recently died, to one of his papers.²¹ I shall conclude with the following statement: the history of physics during the last decades can be identified with the emergence of invariance and the change of scientific thought from substantial conservation to formal invariance.

²¹ F. Bailly, "About the emergence of invariances in physics: from substantial conservation to formal invariance", in *Quantum Mechanics, Mathematics, Cognition and Action*, M. Mugur-Schächter and A. van der Merwe, eds. Dordrecht and Kluwer, 2002.